Distributed Optimization with Pairwise Constraints and its Application to Multi-robot Path Planning

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Problem definition:

(Goal directed navigation of N **heterogeneous robots**)

$$\{\pi_1^*, \ldots, \pi_N^*\} = \operatorname{argmin}_{\pi_1 \ldots \pi_N} \sum_{j=1\ldots N} c(\pi_j)$$

$$\text{s.t.} \quad \Omega_{ij}(\pi_i, \pi_j) = 0$$

(e.g., time-parametrized distance constraint)

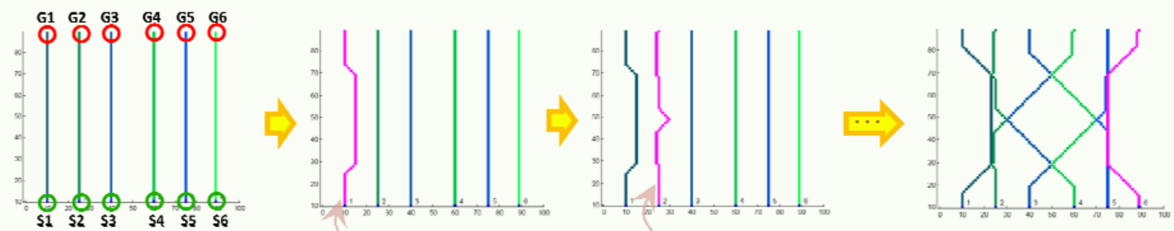
Unconstrained solution

Optimal plan satisfying communication constraints

Planning with extended rendezvous (constraints marked by yellow)

Subproblem: $\pi_i^{iter} = \arg\min_{\pi_i} c(\pi_i) + \sum_{j=1\ldots N, j\neq i} w_{i,j}^{iter} \cdot \Omega(\pi_i, \pi_j)$

Six robots planning iteratively to satisfy ***rendezvous constraints*** in an ***empty environment***:



Iteration (***k***) = ***0***

Iteration (***k***) = ***1*** Planning robot (***r***) = ***1***

Iteration (***k***) = ***2*** Planning robot (***r***) = ***2***

Iteration (***k***) = ***12*** Final converged solution satisfying constraints

Challenges – a very large constrained optimization problem

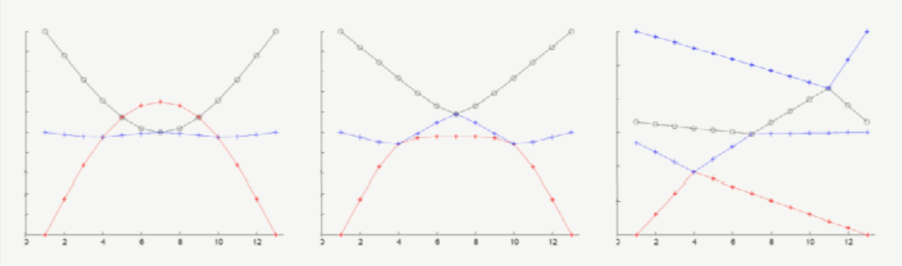
- Size of joint state space **increases exponentially** with N
- **Coupling** due to constraints
- Need of **fast optimal plan** as well as guarantees
- Cluttered, non-trivial environment

Approach – iterative planning in individual state-space, with guarantees

- **Distributed planning**
  a. Model constraints as penalty added to the objective — *soft constraints*.
  b. Iterate over the robots, *gradually increasing* the penalty weights
- *Fast, Efficient*; Provably *Complete, Optimal*.
- Discretization of environment into a graph

Results

An exact implementation



(a) Symmetric case: $\alpha = 0, \beta = 1$ (b) Symmetric case: $\alpha = 1, \beta = 0.2$ (c) An asymmetric case: $\alpha = 1, \beta = 0$

Planning with tasks and constraints

Heterogeneous agents performing complex tasks in 3D: