Flow Induced Vibration

Project Progress Report

Date: 16th November, 2005

Submitted by

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Certificate

This is to certify that the report entitled “Flow Induced Vibration” submitted by Subhrajit Bhattacharya to the Department of Mechanical Engineering, IIT Kharagpur, is a bona fide record of work carried out under my supervision and guidance.

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1. Introduction:

The problem of fluid-structure interaction is encountered in various Engineering applications. The present problem deals with vibrations induced into structures due to flow taking place on its surface, and hence analyzing the stability of the flow. Though the present approach to the problem is grossly simplified and no substantial simulation or numerical solution have yet been deduced, the works done till now have great potential in applications like flow over underwater vehicle, etc.

2. Problem definition and aim:

Our present aim is to analyze the stability of fluid flows over a flexible flat surface. By the term 'stability' we mean that we try to determine the critical Reynolds number for a given flow profile over the structure. We will consider flexibility of the plate and frame the corresponding equations. However for the purpose of determining the numerical solution we have presently investigated only the case of flow over a rigid surface to check if we obtain the standard results for stability of parallel flow over rigid surfaces available in standard text.

3. Origin of Turbulence and ways to analyze stability of a flow:

Flow induced vibration is caused in structures by forcing due to time variant pressure acting on the surface of the structure. If the flow is imagined to be a linear superposition of a steady state laminar flow and a perturbation flow field, the laminar flow won't cause the forcing on the structure as it acts as a time invariant pressure on the structure's surface. It is the perturbation flow field that varies with time and hence produces forcing on the structure's surface.

This same perturbation flow field is the sole cause of Turbulence in the flow. The origin of the perturbation flow field may be something like a very small disturbance caused in the laminar flow field. A flow is said to be stable if for any small initial disturbance (i.e. perturbation) added to the laminar flow field, the perturbation flow field gradually dies down with time. It will be termed as a Turbulent flow if the perturbation flow field gets magnified with time. Hence our primary approach will be to investigate the nature of variation of a perturbation flow field with time.

4. The Orr-Somerfeld equation:

The two-dimensional incompressible flow over the plate is governed by the Navier Stoke's equations,

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}F_X-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (1a)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}F_Y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (1b)$$

and the continuity equation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2)$$

Let us consider a steady-state laminar flow over an infinitely long plate.

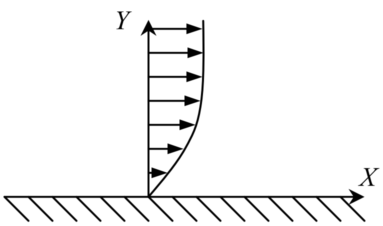

fig - 1

For this flow,

$$\left.\begin{aligned} u &= U(y) \\ v &= 0 \\ p &= P(x) \end{aligned}\right\} \quad (3)$$

Now let the perturbation field be described by the perturbation components denoted by a 'prime' upon the steady-state laminar variables.
Hence the final flow field will be described by,

$$\left.\begin{aligned} u &= U(y) + u'(x, y, t) \\ v &= v'(x, y, t) \\ p &= P(x) + p'(x, y, t) \end{aligned}\right\} \quad (4)$$

Now, both the steady-state laminar field (3) and the superposed field (4) satisfies equation (1) and (2). Hence by substituting them in (1) and (2) and performing some simplification, we obtain the equations governing the perturbation flow field,

$$\frac{\partial u'}{\partial t}+U\frac{\partial u'}{\partial x}+v'\frac{\partial U}{\partial y}+\frac{1}{\rho}\frac{\partial p'}{\partial x}=\nu\,\nabla^2 u' \quad (5a)$$

$$\frac{\partial v'}{\partial t}+U\frac{\partial v'}{\partial x}+\frac{1}{\rho}\frac{\partial p'}{\partial y}=\nu\,\nabla^2 v' \quad (5b)$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0 \quad (5c)$$

From these equations, with a known velocity profile $U(y)$, we may obtain solutions for u', v', and p'. Our present aim will be to determine for a given velocity profile $U(y)$, the coefficient of viscosity ν and the boundary & initial conditions of the perturbation fields, whether or not the perturbation components die down with time.

In order to satisfy eqn. (5c), we define a stream function $\psi(x, y, t)$ such that

$$u' = \frac{\partial \psi}{\partial y} \text{ and } v' = -\frac{\partial \psi}{\partial x} \tag{6}$$

It is now assumed that the disturbances, and hence ψ is superposition of several periodic disturbances (periodic in *x*) propagating along the direction of flow.
Hence, we substitute,

$$\psi(x, y, t) = \phi(y)\, e^{i(\alpha x - \beta t)} \tag{7}$$

ψ is said to be periodic on *x* with frequency α and wavelength $\lambda_X = \dfrac{2\pi}{\alpha}$.
Though α can be assumed to be real, β being the time frequency should be assumed to be complex in order to keep the possibility of non-periodic magnification or decay of ψ with time.
The final ψ will be superposition of all the solutions of ψ.
We define the complex velocity of propagation of the disturbance as

$$c = \frac{\beta}{\alpha} = c_r + i\, c_i \tag{8}$$

$\therefore \psi = \phi(y)\, e^{\alpha[c_i t + i(x - c_r t)]}$
It may be noted that for $c_i < 0$, the solution of ψ dies down with time, and hence so does the perturbation components. Thus the flow is stable for $c_i < 0$ and tends to become turbulent for $c_i > 0$.

We put u' and v' in terms of ϕ, α and β in (5a) and (5b) and eliminate p' to obtain a single equation. We non-dimensionalize the equations by redefining *y* as $\dfrac{y}{\delta}$, *U* as $\dfrac{U}{U_m}$, *c* as $\dfrac{c}{U_m}$, where U_m is the free-stream velocity of the flow and δ is the boundary layer thickness. The non-dimensionalized equation hence obtained is called the Orr-Sommerfeld equation:

$$(U - c)(\phi'' - \alpha^2 \phi) - U''\phi = -\frac{i}{\alpha R}\left(\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi\right) \tag{9}$$

where, $R = \dfrac{U_m \delta}{\nu}$ is the Reynolds Number.

A trivial solution to eqn.(9) is $\phi = 0$. For non-trivial solution, for a given α and *R*, we obtain an eigenvalue of *c* and the corresponding eigenfunction ϕ.
Hence our immediate target is to find an eigenvalue solution of (9).

5. Boundary conditions for the Orr-Sommerfeld equation:

A. Boundary conditions for flow over a rigid, static plate:

For this case,

At $y = 0$,

$v' = 0$ and $u' = 0$

$\Rightarrow \quad \phi(0) = 0$ and $\phi'(0) = 0$

At $y \to \infty$,

$v' = 0$ and $u' = 0$

$\Rightarrow \quad \phi(\infty) = 0$ and $\phi'(\infty) = 0$

B. Boundary conditions for flow over a flexible plate modeled as a beam:

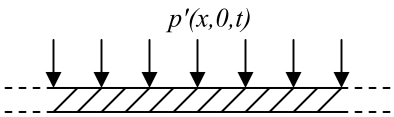

fig - 2

We model the plate as infinite flexible beam. Hence, the differential equation governing the motion of the beam is given by,

$$\lambda \frac{\partial^2 w}{\partial t^2} = -p'(x,0,t) - EI \frac{\partial^4 w}{\partial x^4} \tag{10}$$

Where, w denotes the displacement of the beam in Y direction, λ is mass per unit length of the beam. The forcing on the beam is due to the time-variant perturbation pressure.
It is to be noted in this case that equations (5), (9) and (10) gets coupled.
One way of solving the equation will be as follows.
As p' is of the form of $e^{i(\alpha x - \beta t)}$, we may write for eqn. (10),

$$w(x,t) = k\, e^{i(\alpha x - \beta t)} \tag{11}$$

Substituting this w in (10) and hence substituting the hence obtained $p'(x,0,t)$ into (5a), and Substituting v' in terms of ϕ in the same (5a) equation, all at $y = 0$, we obtain,

$$k = \frac{\rho\left[i\,\alpha U'(0)\phi(0) - (\alpha^2 \nu - i\,\beta)\phi'(0) + \nu\phi''(0)\right]}{i\,\alpha\left(\lambda\beta^2 - EI\alpha^4\right)} \tag{12}$$

Hence the boundary conditions become,

At y = 0,

$v' = \dot{w}$ and $u' = 0$

$\Rightarrow \quad \phi(0) = -\dfrac{\beta}{\alpha}\bar{k}$ and $\phi'(0) = 0$, where $\bar{k}$ is the non-dimensionalized k.

At $y \to \infty$,

$v' = 0$ and $u' = 0$

$\Rightarrow \quad \phi(\infty) = 0$ and $\phi'(\infty) = 0$

Hence, as it can be seen, one of the boundary conditions is a bit more complex involving ϕ, ϕ', ϕ'' and c.

The numerical technique for solving the Orr-Sommerfeld equation for the eigenvalues and eigenfunctions in either of the cases will remain similar, except that the boundary conditions are modified. However till date we have only investigated the case 'A', i.e. the case of flow over a rigid, static plate.

6. Numerical methods attempted for solving the Orr-Sommerfeld equation:

A. Galerkin's Method:

This method, though can handle the case of flow over a rigid, static plate satisfactorily, its application in solving the case of flexible plate is difficult.
The method for the case of flow over a rigid, static plate is described below in brief.

We denote eqn.(9) by $\Gamma(\phi) = 0$, where Γ denotes the operator.
We write ϕ as a linear superposition of several functions ϕ_i that satisfy the boundary conditions given in 5.*A*. Such functions were chosen to be of the form, $\phi_i = y^{\eta} e^{-\mu y}$.
Hence we write,

$$\phi(y) = \sum_{i=1}^{n} \xi_i \, \phi_i(y)$$

We now define an error,

$$e(y) = \Gamma\left(\sum_{i=1}^{n} \xi_i \, \phi_i(y) \right) \tag{13}$$

We need to choose the values of ξ_i in such a way that this error is minimized. We do that by solving the set of *n* equations,

$$\langle e, \phi_i \rangle = 0, \; i = 1 \text{ to } n \tag{14}$$

where, $\langle \chi_i, \chi_j \rangle$ denotes the inner product given by,

$$\langle e, \phi_i \rangle = \int_0^{\infty} e \, \phi_i \, dy$$

The *n* equations in (14) have ξ_i as the unknowns and can be represented in the matrix form as,

$$[M] \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_n \end{bmatrix} = 0 \tag{15}$$

Where the matrix [*M*] contains the unknown *c*.
For non-trivial solution of this equation, we must have,

$$\left| [M] \right| = 0 \tag{16}$$

In general we'll obtain n solutions for c. We should chose the one for which the ξ_i s are such that $\Gamma\left(\sum_{i=1}^{n}\xi_i\,\phi_i(y)\right)$ is minimum.

This method was implemented in Mathematia 5.1 and a rather unsatisfactory result was obtained probably due to the following reasons:

i. Due to limitation of computational power, n had to be limited to 4 in order to get a satisfactory and accurate integration value of the inner product and solution of c from (16).
ii. The choice of η and μ in choosing the functional forms were done arbitrarily.

We determined the eigenvalues c for different α and R and plotted the contour in the α-R plane for which $c_i = 0$. This contour will mark the margin between the stable and unstable zones of α and R. However the contour $c_i = 0$ could not be found satisfactorily in the fist quadrant of α-R plane. But on plotting a contour plot of c_i , the following was obtained. The plot demonstrates that the basic shape of the standard results is being approached by the solution:

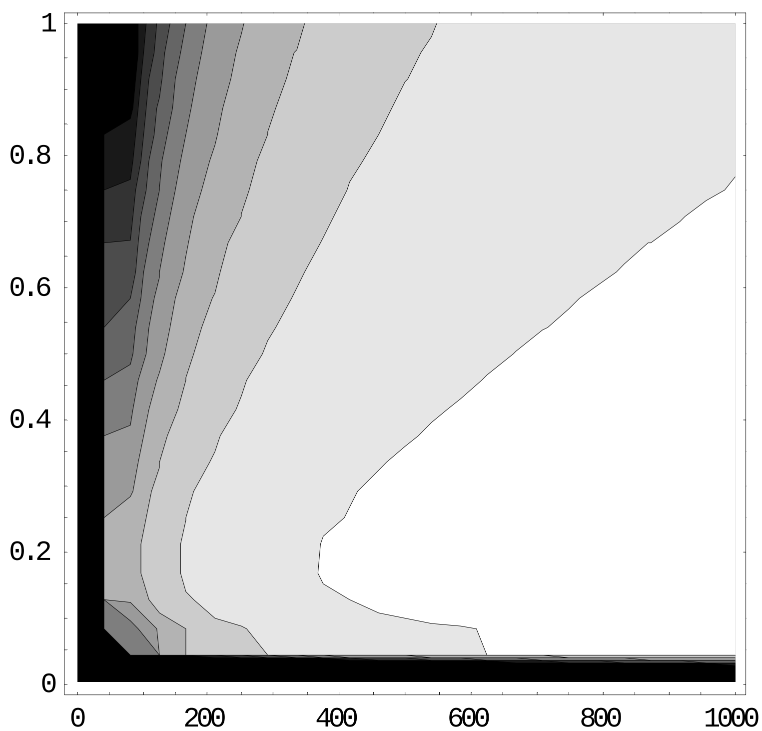

fig - 3

The horizontal axis denotes R and the vertical axis denotes α.

B. 'Automated search of eigenvalues' – Integration by Runge-Kutta:

This method primarily proposed by Betchov & Criminale [2], deals the regions above and below the boundary layer separately. We first investigate the Orr-Sommerfeld equation (9) for $y > 1$. In this region, the non-dimensionalised velocity is $U = 1$. Hence the equation is modified as,

$$(1-c)(\phi''-\alpha^2\phi) = -\frac{i}{\alpha R}\left(\phi''''-2\alpha^2\phi''+\alpha^4\phi\right) \text{ for } y > 1 \tag{17}$$

This equation being 4th order linear in ϕ with constant coefficient has simple analytical solution given by,

$$\phi(y) = \sum_{j=1}^{4} A_j e^{p_j y} \tag{18}$$

where, p_j are the roots of,

$$(1-c)\left(p^2-\alpha^2\right) = -\frac{i}{\alpha R}\left(p^2-\alpha^2\right)^2 \tag{19}$$

Hence, $\quad p_1 = \alpha\,, \qquad p_1 = -\alpha$

$$p_3 = \alpha\left[1+i\frac{R}{\alpha}(1-c)\right]^{1/2}, \qquad p_3 = -\alpha\left[1+i\frac{R}{\alpha}(1-c)\right]^{1/2}$$

From boundary condition at ∞, as y→∞, $\phi = 0$ and $\phi' = 0$.
Hence we have, $A_1 = A_3 = 0$ for $y > 1$.
$\therefore \phi(y) = A_2 e^{p_2 y} + A_4 e^{p_4 y}$.

Now we argue that, since for $y > 1$, the solution is a linear superposition of two modes $e^{p_2 y}$ and $e^{p_4 y}$, the solution for $y < 1$ will also be linear superposition of these two same modes. Hence now our task is to find the solution of these two modes in the region $y < 1$. We attain this by performing two integration passes from $y = 1$ to $y = 0$ independently. We used Runge-Kutta method for this numerical integration.

Integration Pass – I:
We start from $y = 1$ with $A_2 = 0$ and $A_4 = 1$(or some other value).
Therefore we take the initial values $\phi(1) = e^{p_4}$, $\phi'(1) = p_4 e^{p_4}$, $\phi''(1) = p_4^2 e^{p_4}$ & $\phi'''(1) = p_4^3 e^{p_4}$ and move on integrating towards $y = 0$.
Let the solution obtained in this process be called $\phi_I(y)$.

Integration Pass – II:
Similarly with $A_2 = 1$(or some other value) and $A_4 = 0$ we obtain the second pass integration $\phi_{II}(y)$.

Hence the final solution is of the form $\phi(y) = a_I\phi_I(y) + a_{II}\phi_{II}(y)$.

From the boundary conditions at $y = 0$,

$$\phi(0) = a_I\phi_I(0) + a_{II}\phi_{II}(0) = 0 \quad \text{and} \quad \phi'(0) = a_I\phi_I'(0) + a_{II}\phi_{II}'(0) = 0,$$

for non-trivial solution of a_I and a_{II}, we must have,

$$\begin{vmatrix} \phi_I(0) & \phi_{II}(0) \\ \phi_I'(0) & \phi_{II}'(0) \end{vmatrix} = 0 \tag{20}$$

It is to be noted that the only unknown in (20) for a given α and R is c. Hence from 20 we obtain the eigenvalue c. The corresponding eigenvector gives a_I and a_{II}, and hence the final solution of $\phi(y)$.

Search for Eigenvalue in c-plane:

However it was not possible to solve c explicitly from (20). Hence we assumed some c and determined the solutions from the two integration passes, $\phi_I(y)$ and $\phi_{II}(y)$.
For those particular solutions we defined,

$$f(c) = \begin{vmatrix} \phi_I(0) & \phi_{II}(0) \\ \phi_I'(0) & \phi_{II}'(0) \end{vmatrix}$$

As we know *f(c)* should converge to 0, we used an iteration scheme [2] as follows.
We start with an arbitrary value of c and some small value of δc and go on updating it using the following iteration,

$$\Delta c = -\lambda\left(\frac{f(c+\delta c)}{f(c)} - 1\right)^{-1}\delta c$$
$$c \leftarrow c + \Delta c$$
$$\delta c \leftarrow \mu\Delta c$$

where, λ = 1, 0.8 or 0.5 depending on whether the convergence is fast, moderate or slow.
and, $\mu = \frac{1}{4}$.
The iteration continues till Δc reaches a substantially small value.

Like before, we once again searched for the contour of $c_i = 0$ in the α-R plane.
The results obtained, though not satisfactory, is described in the following plot of 20 points:

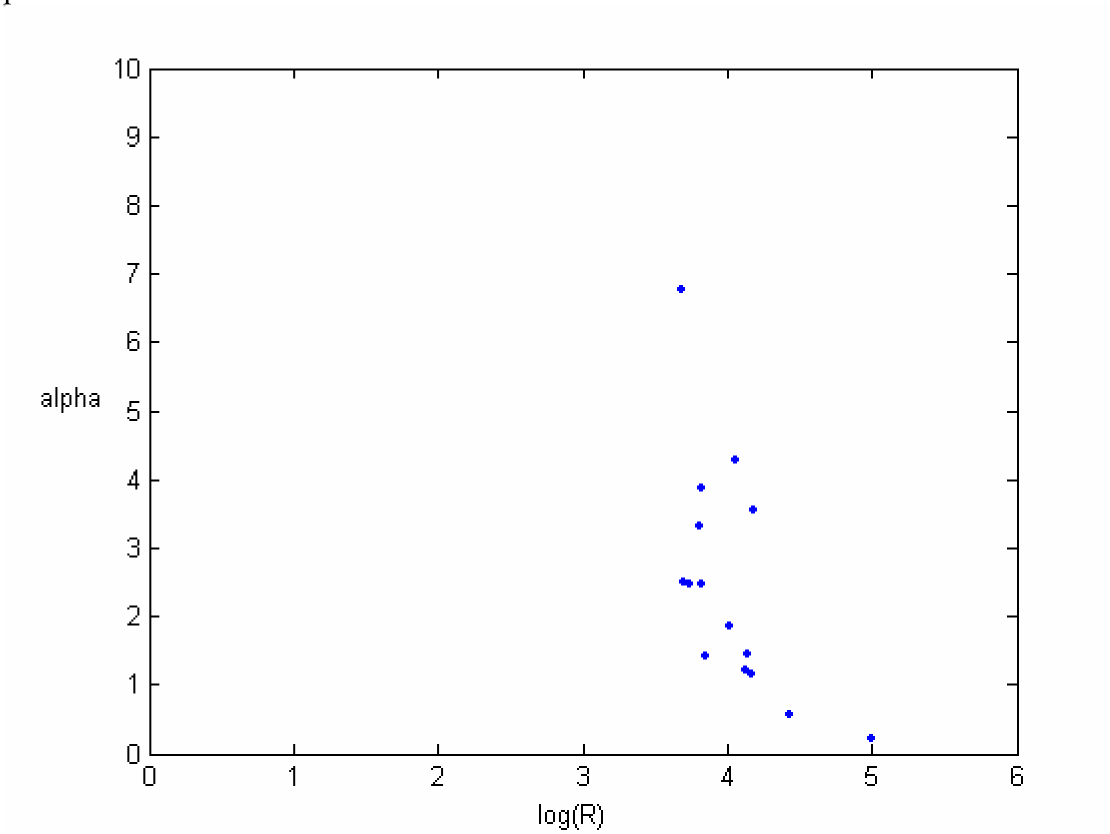

fig - 4

The primary reason for the deviation of points from a single smooth contour is the high oscillation of the solution $\phi_{II}(y)$ as the second numerical integration approaches $y = 0$. This fact is well demonstrated in the following plot of the amplitude of $\phi_{II}(y)$ vs. y for a particular α and R:

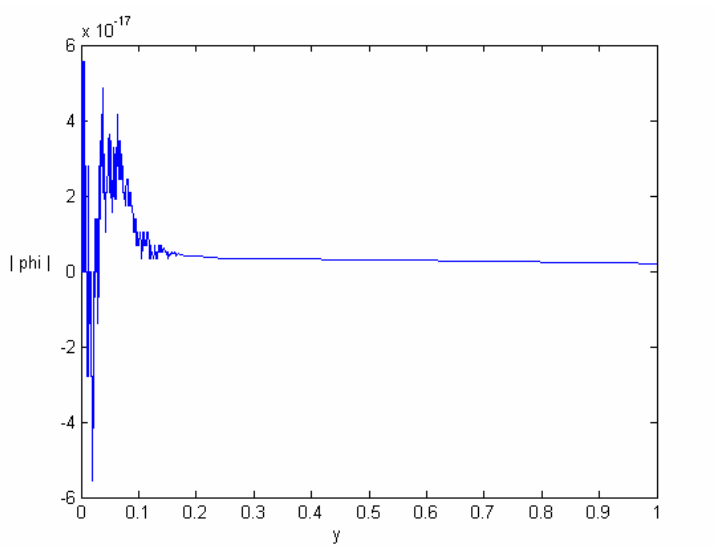

fig - 5

The small amplitude of $\phi_{II}(y)$ is due to the initial choice of a small A_2 for the second Integration pass.

C. Modified second Integration Pass using Stations in between:

This method, as explained by Betchov & Criminale [2], was implemented in order to reduce the oscillation of the second solution. The main principle of this method is based on the choice of some stations in between $y = 1$ and $y = 0$. At these stations the second integration is paused and it is updated by linearly combining with the first integration $\phi_I(y)$ to make $A_4 = 0$.

We are presently working on this technique and hope to obtain some satisfactory result very soon. Once we obtain a solution for the case of flow over a rigid, static plate, we'll attempt the solution for the case of flow over a flexible plate (modeled as an infinite beam) on the similar line.

References:

[1] Dr. Hermann Schlichting, *Boundary Layer Theory*, McGraw Hill, 1968.

[2] Robert Betchov, William O. Criminale, Jr., *Stability of Parallel Flows*, Academic Press, 1967.